

Title: Fixed-parameter tractability, definability, and model checking

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Abstract: In this article, we study parameterized complexity theory from the perspective of logic, or more specifically, descriptive complexity theory.

We propose to consider parameterized *model-checking* problems for various fragments of first-order logic as generic parameterized problems and show how this approach can be useful in studying both fixed-parameter tractability and intractability. For example, we establish the equivalence between the model-checking for existential first-order logic, the homomorphism problem for relational structures, and the substructure isomorphism problem. Our main tractability result shows that model-checking for first-order formulas is fixed-parameter tractable when restricted to a class of input structures with an excluded minor. On the intractability side, for every $t \geq 0$ we prove an equivalence between model-checking for first-order formulas with t quantifier alternations and the parameterized halting problem for alternating Turing machines with t alternations. We discuss the close connection between this *alternation hierarchy* and Downey and Fellows' W-hierarchy.

On a more abstract level, we consider two forms of definability, called *Fagin definability* and *slicewise definability*, that are appropriate for describing parameterized problems. We give a characterization of the class FPT of all fixed-parameter tractable problems in terms of slicewise definability in finite variable least fixed-point logic, which is reminiscent of the Immerman-Vardi Theorem characterizing the class PTIME in terms of definability in least fixed-point logic.

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