Title: Definability and descriptive complexity on databases of bounded tree-width

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Abstract: We study the expressive power of various query languages on relational databases of bounded tree-width.

Our first theorem says that fixed-point logic with counting captures polynomial time on classes of databases of bounded tree-width. This result should be seen on the background of an important open question of Chandra and Harel (1982) asking whether there is a query language capturing polynomial time on unordered databases. Our theorem is a further step in a larger project of extending the scope of databases on which polynomial time can be captured by reasonable query languages.

We then prove a general definability theorem stating that each query on a class of databases of bounded tree-width which is definable in monadic second-order logic is also definable in fixed-point logic (or datalog). Furthermore, for each $k \ge 1$ the class of databases of tree-width at most k is definable in fixed-point logic. These results have some remarkable consequences concerning the definability of certain classes of graphs.

Finally, we show that each database of tree-width at most k can be characterized up to isomorphism in the language $C^{(k+3)}$, the $(k+3)$-variable fragment of first-order logic with counting.