Title: The parameterized complexity of counting problems

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Abstract: We develop a parameterized complexity theory for counting problems. As the basis of this theory, we introduce a hierarchy of parameterized counting complexity classes W[t], for $t > 0$, that corresponds to Downey and Fellows's W-hierarchy and show that a few central W-completeness results for decision problems translate to #W-completeness results for the corresponding counting problems.

Counting complexity gets interesting with problems whose decision version is tractable, but whose counting version is hard. Our main result states that counting cycles and paths of length k in both directed and undirected graphs, parameterized by k, is #W[1]-complete. This makes it highly unlikely that any of these problems is fixed-parameter tractable, even though their decision versions are fixed-parameter tractable. More explicitly, our result shows that most likely there is no $f(k)n^c$-algorithm for counting cycles or paths of length k in a graph of size n for any computable function f and constant c, even though there is a $2^{O(k)}n^{2.376}$ algorithm for finding a cycle or path of length k.

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